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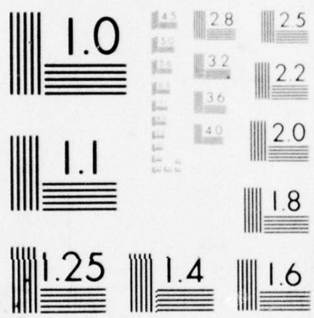

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UCLA-ENG-7726
DECEMBER 1976

FINAL REPORT ON COMMUNICATION METHODS

K. YAO

Principal Investigator – K. YAO

UCLA • SCHOOL OF ENGINEERING AND APPLIED SCIENCE

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20. ABSTRACT: A final report on research activities of ONR contract "Communication Methods", covering the period of 1972 to 1976, is presented. Topics of research include digital and analog communication in the presence of interferences; coding of information sources; and models of stochastic processes for signals and noises. A list of research publications and a list of personnel supported in this period are given. A summary of research activities and their relevance are also included.

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Abstract

A final report on research activities of ONR contract "Communiication Methods", covering the period of 1972 to 1976, is presented. Topics of research include digital and analog communication in the presence of interferences;coding of information sources; and models of stochastic processes for signals and noises. A list of research publications and a list of personnel supported in this period are given. A summary of research activities and their relevance are also included.

I. INTRODUCTION

In this technical report entitled, "Final Report on Communication Methods", research activities covering the period from 1972 to 1976 are described. These investigations were conducted under the support of the Office of Naval Research, Contracts N00014-69-A-0200-4043 and N00014-75-C-0528, and Task NR-042-285. Dr. B.J. McDonald, Director of the Statistics and Probability Program, was the scientific officer in charge of these contracts.

In Section II, a List of Research Publications that appeared in the period 1972-1977 is given. In Section III, a List of Personnel Supported in the period 1972-1976 is presented. In Section IV, a Summary of Research Activities and their relevance are discussed.

II. LIST OF RESEARCH PUBLICATIONS

The following research publications, which appeared in the period 1972-1977, are listed in chronological order.

1. K. Yao, "A Compact Form of Error Expression for Pulse Communication System with Intersymbol Interference," Conference Record of the International Conference on Communications, pp. 46.11-46.14, June 1972.
2. W.D. Hopkins, Structural Properties of Rate Distortion Functions, Ph.D. thesis, UCLA, June 1972. Technical Monograph, Dept. of System Science, UCLA, June 1972.
3. K. Yao, "On Minimum Average Probability of Errors Expression for a Binary Pulse-Communication System with Intersymbol Interference," IEEE Transactions on Information Theory, Vol. IT-18, pp. 528-531, July 1972.
4. J.H. Slaton, Optimum Sonar Performance in the Presence of Reverberation and Noise, Ph.D. thesis, UCLA, Sept. 1972. Technical Monograph, Dept. of System Science, UCLA, Sept. 1972.
5. J.H. Slaton and K. Yao, "Optimum Sonar Signal Design for Matched Filter Detectors," Technical Monograph, Dept. of System Science, UCLA, Sept. 1972.
6. J.H. Slaton and K. Yao, "The Amplitude and Spectrum of Surface Reverberation as Determined from Surface Resonance Theory," Technical Monograph, Dept. of System Science, UCLA, Sept. 1972.
7. J.W. Collier, Numerical Study of Minimum Probability of Error Expression in a Digital Communication System with Intersymbol Interference, M.S. thesis, UCLA, Dec. 1972. UCLA Technical Report, Eng.-7327, Jan. 1973.
8. H. Tan and K. Yao, "Information Rates of Countably Infinite Alphabet Memoryless Sources," Proceedings of the Seventh Annual Princeton Conference on Information Sciences and Systems, March 1973, p. 249 (Abstract).
9. K. Yao and L.B. Milstein, "Maximum Likelihood Receiver for Binary Signals with Intersymbol Interference in Gaussian Noise," Conference Record of the International Conference on Communications, June 1973, pp. 141-144.

10. H. Tan, On the Evaluation of Rate Distortion Functions, Ph.D. thesis, UCLA, June 1973. UCLA Technical Report, Eng.-7356, July 1973.

11. H. Tan and K. Yao, "Information Rate of a Memoryless Gaussian Source under an Absolute Magnitude Criterion," Abstracts of Papers of the 1973 International Symposium on Information Theory, June 1973, p. B3-3.

12. K. Yao, "A Representation Theorem and Its Applications to Spherically-Invariant Random Processes," IEEE Transactions on Information Theory, Vol. IT-19, September 1973, pp. 600-608.

13. K. Yao and H. Tan, "Some Comments on the Generalized Lower Bound for Stationary Finite-Alphabet Sources with Memory," IEEE Transactions on Information Theory, Vol. IT-19, November 1973, pp. 815-817.

14. K. Yao, "Series Expansions for Random Processes on a LCAG," Proceedings of the Eighth Annual Princeton Conference on Information Sciences and Systems, March 1974, p. 58.

15. E. Shomash and K. Yao, "On the Structure and Performance of a Linear Decision Feedback Equalizer Based on the Minimum Error Probability Criterion," Conference Record of the International Conference on Communications, June 1974, pp. 25F.1-5.

16. J. Kawamura, Adaptive Detection in the Presence of Unknown Noise, Ph.D. thesis, UCLA June 1974. Technical Monograph, Dept. of System Science, UCLA, June 1974.

17. K. Yao, "Properties of Rate-Distortion Functions, Part 2: Rate-Distortion Functions with Absolute Value Distortion Measure," in Source Coding/Ergodic Theory Research Seminar Lecture Notes, Ed. by R.M. Gray, Stanford University Tech.Report No. 6503-2, September 1974, pp. 93-101.

18. K. Yao, "Series Expansions and Applications of Second Order Processes on a Locally Compact Abelian Group," Abstractsof Papers of the 1974 International Symposium on Information Theory, October 1974, pp. 24-25.

19. H. Tan and K. Yao, "Rate-Distortion Functions of Countably-Infinite Alphabet Memoryless Sources," Abstracts of Papers of the 1974 International Symposium on Information Theory, October 1974, pp. 63-64.

20. I. Rubin and K. Yao, "A Distortion-Measure Sensitivity Analysis of the Rate-Distortion Function," Proceedings of the Eighth Hawaii International Conference on System Sciences, January 1975, pp. 83-85.

21. H. Tan and K. Yao, "Evaluation of Rate-Distortion Functions for a Class of I.I.D. Sources under an Absolute Magnitude Criterion," IEEE Transactions on Information Theory, Vo. IT-21, January 1975, pp. 59-64.

22. R.M. Tobin and K. Yao, "Error Bounds for Digital Communication Systems in the Presence of Gaussian Noise and Cochannel Interference," Proceedings of the 1975 Conference on Information Sciences and Systems, April 1975, p. 196 (Abstract).

23. H. Tan and K. Yao, "Rate-Distortion Functions of Countably-Infinite Alphabet Memoryless Sources," Information and Control, Vo. 27, March 1975, pp. 272-288.

24. K. Yao and R.M. Tobin, "A New Class of Upper and Lower Error Bounds for Digital Communication Systems with Intersymbol Interference," Conference Record of the 1975 International Communication Conference, June 1975, pp. 35.11-35.15 .

25. K. Yao, "Encoding of a Continuous Source by Twisted Modulation," Proceedings of the 1975 Allerton Conference on Circuit and System Theory, October 1975, pp. 481-492.

26. K. Yao and L.B. Milstein, "On ML Bit Detection of Binary Signals with Intersymbol Interference in Gaussian Noise," IEEE Transactions on Commuications, Vol. COM-23, September 1975, pp. 971-976.

27. K. Yao and R.M. Tobin, "Moment Space Upper and Lower Error Bounds for Digital Systems with Intersymbol Interference," IEEE Transactions on Information Theory, Vol. IT-22, January 1976, pp. 65-75.

28. T.Y. Yan, Moment Space Error Bounds for Digital Communication Systems with Intersymbol Interference Based on Nth Moment, M.S. thesis, UCLA December 1975. UCLA Technical Report Eng.-7608, January 1976.

29. K. Yao, "Error Probability of Spread Spectrum Multiple Access Communication Systems," Proceedings of the 1976 Conference on Information Sciences and Systems, March 1976, pp. 67-72.

30. K. Yao and H. Tan, "On Rate-Distortion Functions of I.I.D. Sources under an Absolute-Magnitude Criterion," Proceedings of the 1976 Conference on Information Sciences and Systems, March 1976, p. 61 (Abstract).

31. K. Yao and R.M. Tobin, "Multi-Dimensional Moment Space Error Bounds for Digital Systems with Multiple Interferences," Abstracts of Papers of the International Symposium on Information Theory, June 1976, p. 69 (Abstract).

32. K. Yao, "Performance Bounds on Spread Spectrum Multiple Access Communication Systems," Proceedings of the International Telemetry Conference, September 1976, pp. 317-325.

33. K. Yao, "Quadratic-Exponential Moment Error Bounds for Digital Communication Systems," Conference Record of the Tenth Annual Asilomar Conference on Circuits, Systems, and Computers, November 1976, pp. 99-103.

34. R.M. Tobin and K. Yao, "Upper and Lower Error Bounds for Coherent Phase-Shift-Keyed Systems with Cochannel Interference," IEEE Transactions on Communications, Vol. COM-25, February 1977, pp. 281-287.

35. K. Yao, "Error Probability of Asynchronous Spread Spectrum Multiple Access Communication Systems," IEEE Transactions on Communications, Vol. COM-25, Special Issue on Spread Spectrum Communications, Summer 1977 (full-length paper).

36. K. Yao, "Moment Space Error Bounds in Digital Communication Systems," Proceedings of the NATO Advanced Study Institute, Aug. 1977.

37. Data Communication - Intersymbol Interference, Ed. by K. Yao (Dowden, Hutchinson and Ross, publ.) Summer, 1977.

38. K. Yao and H. Tan, "Evaluation of Rate-Distortion Functions for I.I.D. Sources with Support on Subsets of $\mathbb{R}$ under an Absolute-Magnitude Criterion," submitted to the IEEE Transactions on Information Theory.

III. LIST OF PERSONNEL SUPPORTED BY THE CONTRACT

The following people performed work and were partially supported by the contracts during the period 1972 to 1976. The names are listed in alphabetical order.

1. J.W. Collier, M.S. degree, received 1972.
2. W.D. Hopkins, Ph.D. degree, received 1972.
3. M. King, Ph.D. degree candidate, 1976.
4. J. Kawamura, Ph.D. degree, received 1974.
5. S. Reisenfeld, Ph.D. degree candidate, 1975-1976.
6. E. Shomash, Ph.D. degree, received 1973.
7. J.H. Slaton, Ph.D. degree, received 1972.
8. H. Tan, Ph.D. degree, received 1973.
9. R.M. Tobin, Ph.D. degree, received 1976.
10. T.Y. Yan, Ph.D. degree candidate, 1976.
11. K. Yao, Associate Professor and Principal Investigator, 1972-1976.

IV. SUMMARY OF RESEARCH ACTIVITIES

In this section, we present a brief summary of the research activities that have been preformed in the period from 1972 to 1977. The theoretical and practical relevances of these research results are discussed. Our discussions are grouped along the line of three fields of interest. The first field is in digital and analog communication in the presence of interferences. The second field is in coding of information sources. The third field is in models of stochastic processes for signals and noises.

Now, consider our first field of research interest. In an idealized environment, the most basic aspect of statistical communication theory is to decide optimally the presence or absence of a signal in random noise. In practice, due to the presence of limitations in real transmission channels, interferences induced by the signals are generally present. Thus, these interferences, which are random, combine with the ever present additive random channel noise to degrade the performance of the overall system. There is a significant difference in performance for such a system that has been optimized as compared to one that neglects the presence of such interference.

In our sonar and reverberation channel study, from [4] ("Optimum Sonar Performance in the Presence of Reverberation and Noise"), relevant past techniques are reviewed critically and some new practical results are obtained. The modern techniques of detection and signal design are useful to practical sonar problems only if the statistical characteristics of undersea reverberation channels are known. In the past, these characteristics were derived from parameters on an ad-hoc point-scatterer model. In [6] ("The Amplitude and Spectrum of Surface Reverberation as Determined from Surface Resonance

Theory"), we removed the need for these expedients with respect to the surface component of reverberation by developing a direct relation between the reverberation characteristics and the physical, acoustical, and geometrical parameters. Our theoretical results agree well with the limited available experimental data. In [5]("Optimum Signal Design Methods for Sonar Detection"), we presented an optimum signal design procedure for a sonar design which operates in a background of reverberation and noise, and which used a practical matched filter detector. Results from our procedure were compared with the conceptually superior (but much more complicated) reverberation filter. We have shown in practical situations that the reverberation filter may be superior only in a limited range of conditions. Thus, our results are of considerable interest from the practical sonar design point of view.

In any digital communication transmission channel, there is only available a finite bandwidth of transmission frequency. Thus, any finite time-duration pulse representing some digital data upon passage through the transmission channel will have infinite time duration. Consequently, resulting pulses representing adjacent data will cause intersymbol interferences. Consequently, intersymbol interference is an extremely active topic of research and development in communication technology. This is due to the increasing transmission traffic of digital data by commercial and governmental agencies. In [1] and [3] ("A Compact Form of Error Expression for Pulse Communication System with Intersymbol Interference" and "On Minimum Average Probability of Error Expression for a Binary Pulse-Communication System with Intersymbol Interference"), we have obtained a new analytical equation for the optimum coefficients of a linear equalizer under a minimum probability of error criterion. Before the existence of our result, the exact closed-

form expression for the minimum probability of error has been considered to be "nearly impossible to compute" (Lucky, Satz, and Weldon, "Principles of Data Communications," McGraw-Hill, 1968, p.104).

In [15] ("On the Structure and Performance of a Linear Decision Feedback Equalizer Based on the Minimum Error Probability Criterion"), we used the above technique for a linear decision feedback equalizer. A decision feedback equalizer's performance can be considerably better than that of a linear equalizer with only slight increase in complexity. Past work by others on linear and decision feedback equalizers have been based mostly on mean-square or peak distortion criteria. For digital communcation system analysis, the probability of error is of course the true criterion of importance. In practice, equalizers designed on the basis of minimum probability or error criterion are only slightly superior to those based on other criteria. Nevertheless, we feel our work on these equalizers provides a firm foundation for the analysis of these systems. Yet, despite the frequent use of linear and decision-feedback equalizers in practice, it is known that the structure of the optimum equalizer should be nonlinear and based on the maximum-likelihood criterion. In [9] and [26] ("Maximum-Likelihood Receiver for Binary Signals with Intersymbol Interference in Gaussian Noise" and "On ML Bit Detection of Binary Signals with Intersymbol Interference in Gaussian Noise"), we considered such an equalizer. Besides various general basic properties, for the first time we have shown in these papers that it is possible to obtain the performance (i.e., probability of error) of this highly complicated nonlinear receiver by analytical approximation without tedious numerical simulations.

A basic problem in digital communication theory is the evaluation of error probability. In most problems of realistic interest, it is generally

not possible to evaluate it analytically. Of course, one can perform a simulation of the system. Often a simulation is expensive and generally one obtains no real insight into the problem; consequently, one is forced to use various bounding techniques. While standard results of Chebychev inequality and Chernoff bound may be adequate for information theory purposes, where one is generally only interested in asymptotically large samples, these techniques often do not yield adequately tight bounds for practical digital communication systems. We have used results from the theory of moment spaces and have obtained new upper and lower bounds on error probability that are significantly tighter than previously known bounds for several practical digital communication system problems. The basic moment space isomorphism theorem states that, for a random variable of distribution of compact support, the compact body M, defined as the moments of the r.v. with respect to n arbitrary continuous functions, is isomorphic to the convex hull of the curve traced out by the function values of these n functions. While this basic theorem is by Karlin and Shapley, we believe that it has not been used in information theory nor in communication system analysis. In [27] ("Moment Space Theory Upper and Lower Error Bounds for Digital Communication Systems with Intersymbol Interference") and [24] ("A New Class of Upper and Lower Bounds for Digital Communication Systems with Intersymbol Interference"), we considered a linear equalizer, based on any error criterion, and used the above moment space isomorphism theorem. We showed various previously known bounds on error probability can be reduced to some of our special cases. Furthermore, we constructed a class of "exponential kernel" bounds which can be orders of magnitude tighter than previously known bounds. Thus , these bounds are not

only tight but are reasonably simple to use. In [34] ("Error Bounds for Digital Communication Systems in the Presence of Gaussian Noise and Co-channle Interference"), we used the above technique for a class of cochannel satellite and microwave communication link analyses. These problems are of considerable practical importance and our results are again significantly tighter than previously known results.

In recent years, there has been considerable interest in communication systems with spread spectrum and multiple access capabilities. The spread spectrum effect is clearly of use for secure communication purposes. For satellite applications, spread spectrum modulation yields a conceptually simple solution to the requirement of lowering radio frequency interferences of one user by another. On the other hand, multiple access capability is of benefit when many users share the bandwidth and transmission capability of a communication system, or for a satellite communication system with a single wide band repeater. In such situations, a code modulation asynchronous spread spectrum multiple access (SSMA) system is applicable in a network of low-cost mobile ground-based and air-bounced users requiring no network control. The SSMA technique has also been used in multi-user computer communication networks. In any case, for all these and other related SSMA systems, the exact evaluation of error probability has been considered a formidable task. Error probability obtained by a complete simulation of such systems may involve considerable computational cost and need not yield much insight.

In [29] ("Error Probability of Spread Spectrum Multiple Access Communication Systems"), [32] ("Performance Bounds on Spread Spectrum Multiple Access Communication Systems") and [35] ("Error Probability of Asynchronous Spread Spectrum Multiple Access Communication Systems"), we obtained upper and lower

bounds of these spread spectrum multiple access communication systems by using the space technique. Derivations for the second moment, fourth moment, single exponential moment, and multiple exponential moment are given in terms of the partial cross correlations of the codes used in the system. Error bounds based on the use of these moments are obtained. By using a sufficient number of terms in the multiple exponential moment, upper and lower bounds can be made arbitrarily tight. In that case, the error probability equals the multiple exponential moment of the multiple access interference random variable.

It is clear that many other closely related digital communication systems and digital signal processing error probabilities can be bounded using this technique. We believe this moment space bounding technique has significant impact on the theoretical understanding of error probability in digital signal analysis. At the practical computational level, our moment space bounds are reasonably attractive and in all cases are equal to or tighter than any other known error bounds.

In the second field of research interest, we considered coding of information sources. Due to the world-wide "information explosion", there is a real need for data reduction prior to information storage, processing, and transmission. We have been mainly interested in the theoretical aspects of this problem. A rate-distortion function, originated by Shannon, is the minimum information rate needed to specify a source between the input and output, subject to some tolerable distortion. Thus, this quantity is basic to the study of source encoding. In [2] ("Structural Properties of Rate-Distortion Function"), we have shown that the basic concept of "innovation" (i.e., "new information"), originated by Wold and commonly used in linear

and nonlinear filtering theory, is also useful in the evaluation of the rate-distortion function. In general, it is difficult to calculate the rate-distortion function of a source and the standard approach is to calculate the more easily obtainable Shannon lower bound. Often this lower bound is quite close to the actual function and sometimes it is actually equal to it. Up to now, for discrete sources, the Shannon lower bound approach has been applicable only to those with finite alphabets. In [8] ("Rate-Distortion Functions of Countably-Infinite Alphabet Memoryless Sources") and [27] ("Moment Space Upper and Lower Error Bounds for Digital Systems with Intersymbol Interference"), we have shown for the first time that it is possible to use the Shannon lower bound approach for the countably-infinite alphabet sources. We used quite advanced functional analysis tools of the contraction mapping theorem and Schauder fixed-point theorem for this problem. In [11] ("Information Rate of a Memoryless Gaussian Source under an Absolute Magnitude Criterion") and [21] ("Evaluation of Rate-Distortion Functions for a Class of I.I.D. Sources under an Absolute Magnitude Distortion Criterion"), we have shown for the first time that it is possible to evaluate the rate-distortion function exactly for this class of continuous sources, including the Gaussian source, by variational argument when the Shannon lower bound is never equal to the rate-distortion function. In [17] ("Properties of Rate-Distortion Functions, Part 2: Rate-Distortion Functions with Absolute Value Distortion Measure") and [38] ("Evaluation of Rate-Distortion Functions for I.I.D. Sources with Support on Subsets of $\mathbb{R}$ under an Absolute-Magnitude Criterion"), by using this technique we were able to obtain exactly the rate-distortion function of a uniform source and a Poisson distributed source. In [13] ("Some Comments on the Generalized Shannon Lower Bound for Stationary Finite-Alphabet Sources with Memory"),

we have shown that the most general published results on this topic contained a basic error. We have then shown for the first time that it is possible to establish conditions for the equality of the Shannon lower bound for a class of stationary infinite memory finite-alphabet sources.

In [20] ("A Distortion-Measure Sensitivity Analysis of the Rate-Distortion Function"), we considered a completely new type of problem in which the source statistics are assumed to be known but the appropriate distortion measure to be used is assumed to be not completely specified. Thus, we considered the sensitivity of the rate-distortion function in terms of the distortion measure. This problem is philosophically similar to the class of problems in which the distortion measure is fixed but source statistics are not completely specified. It is clear that, in order to use rate-distortion theory in any practical sense, one must be able to understand both classes of problems since the source statistics are never known completely in advance nor does there exist only one proper distortion measure of interest.

In [25] ("Encoding of a Continuous Source by Twisted Modulation"), the rate-distortion function of a uniformly distributed source under the mean-square error criterion was evaluated by numerical technique. Then the maximum output versus input SNR of an analog communication system with bandwidth expansion for bandlimited i.i.d. uniformly distributed input source was obtained. Some preliminary method of attack on obtaining an upper bound on the mean-square error of the minimum probability of error estimator was also discussed.

In the third field of research interest, we considered models of stochastic processes for signals and noises. In [12] ("A Representation Theorem and its Applications to Spherically-Invariant Random Processes"), we obtained a

basic representation theorem for this class of processes. The n^{th}-order characteristic functions and probability densities of these processes have been given in an explicit manner. This result is important to probability theory since the classes of n^{th}-order probability densities given in a constructive manner are quite limited. Furthermore, this representation theorem showed a fundamental relationship between s.i.r.p. and the random mixture of Gaussian processes. Our results are more general than, and include, all previously known results on this topic (e.g., Vershik of U.S.S.R., Picinbono of France, Blake of Canada, Copper of U.S.A., etc.). It also turned out that very low frequency (VLF) atmospheric noise, relevant to submarine communications, can be modeled by mixtures of Gaussian noise.

The theory of wide-sense stationary stochastic processes indexed on the integers and the real line has been well developed. Many results for wide-sense stationary processes are still valid when the parameter set is taken from an abstract locally-compact abelian group (LCAG). On the other hand, sampling expansions used in imformation and communication theory modeling, which were originally derived for bandlimited deterministic signals, have been extended in our studies to bandlimited wide-sense stationarly stochastic processes on the real line. In [14] ("Series Expansions for Random Processes on a LCAG") and [18] ("Series Expansions and Applications of Second Order Processes on a Locally Compact Abelian Group"), we obtained sampling expansions for bandlimited signals and wide-sense stationary stochastic processes and Karhunen-Loeve type expansions on an abstract LCAG. Furthermore, reproducing kernel Hilbert space techniques, originally proposed by Loeve (1955) and Parzen (1960) for wide-sense stationary stochastic processes on the real line, and by Yao ("Applications of RKHS -- Bandlimited

Signal Modals," Information and Control, 1967) for deterministic signals, have also been used in an abstract LCAG setting. These expansion results on LCAG are not only of academic interest, but shed some light on Fourier-Walsh sampling expansions used in some practical data reduction schemes.

In conclusion, we have used various advanced mathematical techniques in probability, statistics, and functional analysis to study various theoretically and practically oriented problems in communication and information theories . Our results have contributed to the growth of theoretical understanding of these problems. Our results have also contributed significantly to the practical evaluation of communication system performance. In this contract period, over a dozen people including graduate students, faculty associates, and the principal investigator have been supported. Of our Ph.D. graduates: Dr. J. Slaton is a highly regarded torpedo designer at Naval Undersea Center at San Diego; Dr. H. Tan is an Assistant Professor at Princeton University; Dr. Shamash and Dr. Tobin are with Hughes Aircraft Company at Los Angeles; Dr. D. Hopkins is at IBM at Thousand Oaks, California; Dr. J. Kawamura is at TRW Systems at Redondo Beach, California. This research work, performed during their thesis study, has contributed significantly to their professional growth. In this period, the principal investigator has also done consulting work at Bunker Ramo on a sea-return cable high data communication system sponsored by the Department of Navy and at Hughes Aircraft Company on radar signal analysis for the Naval F14 aircraft. The research activities performed under this contract also contributed significantly to the successful completion of these tasks.